

Hadron resonance gas and nonperturbative QCD vacuum at finite temperature.

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Abstract

We study the nonperturbative QCD vacuum with two light quarks at finite temperature in the framework of hadron resonance gas. Temperature dependence of the quark and gluon condensates in the confined phase are obtained. We demonstrate that the quark condensate and one half (chromo-electric component) of gluon condensate evaporate at the same temperature, which corresponds to the temperature of quark-hadron phase transition. Critical temperature is $T_c \simeq 190$ MeV when temperature shift of hadron masses is taken into account.

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1. As well known, QCD at finite temperature undergoes a phase transition from hadron phase, characterized by confinement and chiral symmetry breaking, to the phase of hot quark-gluon matter. At the critical point T_c where the phase transition occurs the behavior of the thermodynamic properties of the system, such as energy density ε , specific heat, non-ideality $(\varepsilon - 3P)/T^4$, etc., is drastically changed. More than that, the phase transition in QCD is characterized by the radical rearrangement of the non-perturbative quark-gluon vacuum.

Lattice calculations for finite temperature QCD show that the deconfinement and chiral invariance restoration take place at the same temperature, and for the case of two light quarks ($N_f = 2$) critical temperature is in the interval $T_c \sim 175 \div 190$ MeV [1, 2]. From the studies of QCD on the lattice and from experimental data on high energy collisions it also follows that energy density of the system at quark-hadron phase transition is of the order of $\varepsilon_c \sim 1 \div 1.5$ GeV/fm³.

Recently it was demonstrated in the lattice simulations that for gauge $SU(3)$ theory without quarks and for QCD with $N_f = 2$ electric component of gluon condensate is strongly suppressed above critical temperature T_c , while magnetic component even slightly

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grows with temperature [3]. Also, gluon condensate at finite temperature in $SU(2)$ lattice gauge theory was studied in the earlier paper [4]. It was shown there that approximately one half of the gluon condensate does not vanish above T_c .

These results are in line with theoretical predictions of the deconfining phase transition within the "evaporation model" [5] approach. Within the framework of the effective dilaton Lagrangian at finite temperature the temperature dependence of the gluon condensate and its discontinuity at $T = T_c$ in pure-gluon QCD was studied in [6]. Later in the paper [7] temperature dependence of the gauge invariant bilocal correlator of chromomagnetic fields and spatial string tension $\sigma_s(T)$ were found analytically. It was obtained that the chromo-magnetic condensate at $T < 2T_c$ grows slowly as temperature increases, $\langle H^2 \rangle_T = \langle H^2 \rangle_0 \coth(M/2T)$, where $M = 1/\xi_m$ is the inverse magnetic correlation length, which does not depend on temperature at $T < 1.5T_c$ [3]. In the region $T > 2T_c$ magnetic correlator amplitude grows, $\langle H^2 \rangle_T \propto g^8(T)T^4$, and correlation length decreases, $\xi_m(T) \propto 1/(g^2(T)T)$, with increasing temperature. This behavior of magnetic correlator [7] explains magnetic confinement in the framework of the stochastic vacuum model. Obtained temperature dependence of the spatial string tension is in perfect agreement with lattice data [8] in all temperature regions.

Recently the connection between deconfining and chiral phase transitions in QCD was studied in [9] in the framework of effective models which take into account Polyakov loops dynamics. Authors discuss that phase transitions take place at the same temperature. Also the deconfining transition in effective model of the Yang-Mills theory with non-order parameter fields was considered in [10].

Thus, taking into account above listed facts, one has to obtain in the framework of single approach that in QCD with $N_f = 2$ at the critical point $T_c \sim 175 \div 190$ MeV energy density reaches value $\varepsilon_c \sim 1 \div 1.5$ GeV/fm³, quark condensate $\langle \bar{q}q \rangle_T$ vanishes, and only one half of gluon condensate (chromo-electric component, responsible for the formation of string and confinement) "evaporates", which is required to retain magnetic confinement.

In this paper we study temperature properties of quark and gluon condensates in the approach, based on description of the confined phase as hadron resonance gas¹. We demonstrate, that all above listed phenomena can be quantitatively explained in the framework of this approach if one takes into account temperature shift of hadron masses.

2. We will consider QCD with two light quarks. Then, knowing pressure in the hadronic phase, $P_h(T)$, and making use of Gell-Mann-Oakes-Renner (GOR) relation, one can find the temperature dependence of quark condensate

$$\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = 1 - \frac{1}{F_\pi^2} \frac{\partial P_h(T)}{\partial m_\pi^2}, \quad (1)$$

where $F_\pi = 93$ MeV is the axial π -meson decay constant. Expression for gluon condensate $\langle G^2 \rangle_T \equiv \langle (gG_{\mu\nu}^a)^2 \rangle_T$ was derived in [12] starting from the renormalization group consideration of anomalous contribution to the energy-momentum tensor in QCD with $N_f = 2$ at finite temperature. Relation connecting gluon condensate with thermodynamical pressure

¹Hadron resonance gas model was proposed by R. Hagedorn [11] for the description of hot strongly-interacting matter.

in QCD is given by [12]

$$\langle G^2 \rangle_T = \langle G^2 \rangle_0 + \frac{32\pi^2}{b} \left(4 - T \frac{\partial}{\partial T} - m_\pi^2 \frac{\partial}{\partial m_\pi^2} \right) P_h(T), \quad (2)$$

where $b = 11N_c/3 - 2N_f/3 = 29/3$. QCD low energy theorems [13] and GOR relation, which relates mass of light quark to the π -meson mass, were used to derive (2). Connection between trace anomaly and thermodynamic pressure in the chiral limit and in pure-gluon QCD was also considered in [14] and [15] correspondingly. Expressions for $\langle \bar{q}q \rangle_T$ and $\langle G^2 \rangle_T$ in QCD with $N_f = 3$ were obtained in [16]. Thus, knowing pressure $P_h(T)$ as a function of temperature and π -meson mass one can find temperature dependence of quark and gluon condensates in the hadronic phase.

In the framework of the Chiral Perturbation Theory (ChPT) the temperature dependence of the quark condensate was first studied in Refs. [17] and at the three-loop level of ChPT in [18]. Using the virial expansion in a gas made of pions, kaons and etas the temperature dependence of the quark condensate in ChPT in two and three flavor cases was studied in [19].

To describe thermodynamics of QCD in the confined phase we make use of hadron resonance gas model. In this approach all thermodynamic properties of the system are determined by the total pressure of relativistic Bose and Fermi gases, which describe temperature excitations of massive hadrons. The motivation of using this method is that it incorporates all essential degrees of freedom of strongly interacting matter. Moreover, the use of hadron resonances spectrum effectively takes into account interactions between stable particles. Description of multiple particle production in heavy ions collisions in the framework of hadron resonance gas [20] leads to good agreement with experimental data.

Thus, pressure in the confined phase is given by

$$\begin{aligned} P_h &= T \sum_i g_i \eta_i \int \frac{d^3p}{(2\pi)^3} \ln (1 + \eta_i e^{-\omega_i/T}), \\ \omega_i &= \sqrt{p^2 + m_i^2}, \\ \eta_i &= \begin{cases} +1, & \text{fermions} \\ -1, & \text{bosons} \end{cases} \end{aligned} \quad (3)$$

where g_i is the spin-isospin degeneracy factor (e.g. $g_\pi = 3$, $g_N = 8$, ...). The energy density $\varepsilon_h = T \partial P_h / \partial T - P_h$ in the hadronic phase is given by

$$\varepsilon_h = \sum_i g_i \int \frac{d^3p}{(2\pi)^3} \frac{\omega_i}{\exp(\omega_i/T) + \eta_i} \quad (4)$$

3. To study condensates in the confined phase quantitatively the knowledge about pressure P_h dependence on light quark mass, or which is the same, on π -meson mass is needed. In the framework of hadron resonance gas model it is equivalent to the knowledge of masses of all resonances as functions of pion mass. This dependence was studied numerically on the lattice, and in the paper [21] five parameters formula, inspired by bag

model, was suggested. At the certain choice of parameters it accurately describes masses of all considered by the authors [21] particles

$$\begin{aligned}
m_i &= N_u a_1 x \sqrt{\sigma} + \frac{m_h}{1 + a_2 x + a_3 x^2 + a_4 x^3 + a_5 x^4}, \\
x &\equiv m_\pi / \sqrt{\sigma}, \\
a_1 &= 0.51, \quad a_2 = a_1 N_u \sqrt{\sigma} / m_h, \\
a_3 &= 0.115, \quad a_4 = -0.0223, \quad a_5 = 0.0028.
\end{aligned}
\tag{5}$$

Here m_h is the physical hadron mass, N_u is the number of light quarks ($N_u = 2$ for mesons, $N_u = 3$ for baryons), $\sigma = (0.42 \text{ GeV})^2$ is the string tension.

Next, it should be taken into account that as temperature increases hadron masses change. In the framework of finite temperature conformally generalized nonlinear sigma model with light and massive hadrons [22] it was shown, that temperature shift of hadron masses can be taken into account by the following substitution

$$\begin{aligned}
m_h &\rightarrow m_h(\chi_T/\chi_0), \quad m_\pi \rightarrow m_\pi \sqrt{\chi_T/\chi_0}, \\
\chi_T/\chi_0 &= (\langle G^2 \rangle_T / \langle G^2 \rangle_0)^{1/4},
\end{aligned}
\tag{6}$$

where χ is the dilaton field. Different as compared to other particles dependence of π -meson mass is the reflection of it's goldstone nature. In the chiral limit, $m_q \rightarrow 0$, presented relation for the hadron masses is a strict consequence of low-energy QCD theorems. Similar to (6) relations for the shift of nucleon mass were obtained using effective dilaton Lagrangian at finite baryon density [23].

4. Formulas (1)-(6) define thermodynamic properties of the system in hadronic phase and allow to calculate quark and gluon condensates in the whole region of temperatures below T_c .

We take into account all hadron states with masses below 2.5 GeV for mesons and 3.0 GeV for baryons. Altogether it amounts to 2078 states (with degeneracy factors g_i taken into account). It is clear, that at low temperatures $T < m_\pi = 140 \text{ MeV}$ main contribution to the thermodynamic quantities will come from thermal excitations of π -mesons, since other states are substantially heavier and are exponentially suppressed with Boltzman factor $\propto \exp\{-m_h/T\}$. However, a great many of heavy states starts playing important role when $T > m_\pi$. In figure 1 pion contribution to the pressure is shown with dash-dotted line. It is seen, that below temperature $T = 120 \text{ MeV}$ pions give main contribution to the P_h . At higher temperatures main contribution comes from all other hadron states. Figure 1 also shows lattice data [24] for the pressure P_h in QCD with $N_f = 2$. One can see that in the region $T < T_c$ hadron resonance gas model with the account for temperature mass shift gives good description of pressure as a function of temperature.

In figure 2 energy density ε_h as a function of temperature is presented. Value of 1 GeV/fm^3 , corresponding to the estimates for energy density at quark-hadron phase transition, is reached at temperature $T \simeq 175 \text{ MeV}$, i.e. in the region of phase transition temperature, as obtained in lattice calculations [25].

Figures 3 and 4 show quark and gluon condensates as functions of temperature. Shaded area in figure 4 corresponds to zero-temperature values of gluon condensate being in the range $\langle G^2 \rangle_0 = (0.5 \div 1.0) \text{ GeV}^4$. It is important that quark condensate goes to zero at the same temperature, where half of gluon condensate evaporates, and if temperature shift of hadron masses is taken into account, this temperature is $T \simeq 190 \text{ MeV}$.

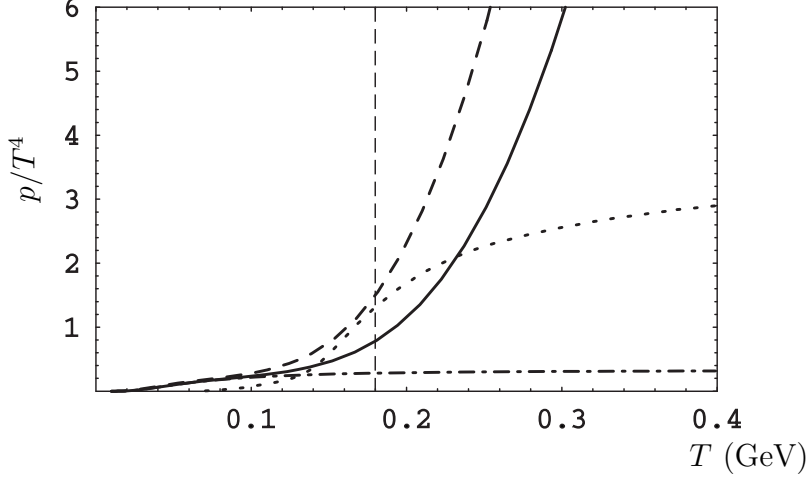


Figure 1: Pressure P_h/T^4 as a function of temperature. Solid line – zero temperature hadron spectrum; dashed line – spectrum with temperature shift taken into account, $\chi_T/\chi_0 = 0.84$; dash-dotted line – pion excitations only. Dotted line – lattice data [24].

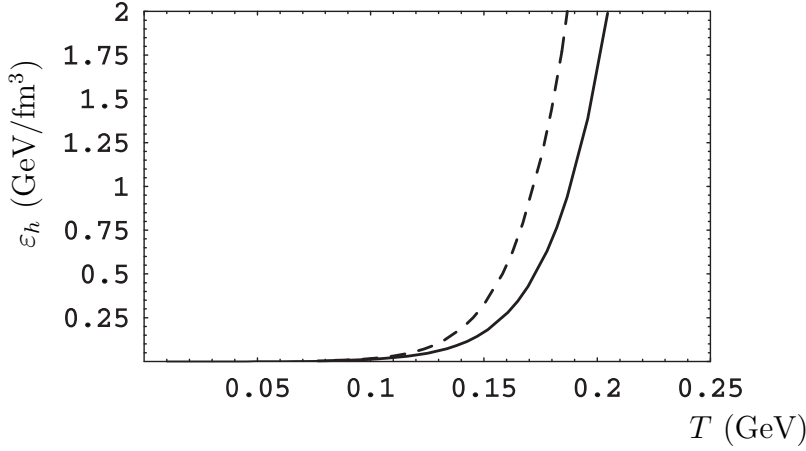


Figure 2: Energy density ε_h as a function of temperature. Solid line – zero temperature hadron spectrum; dashed line – spectrum with temperature shift taken into account, $\chi_T/\chi_0 = 0.84$.

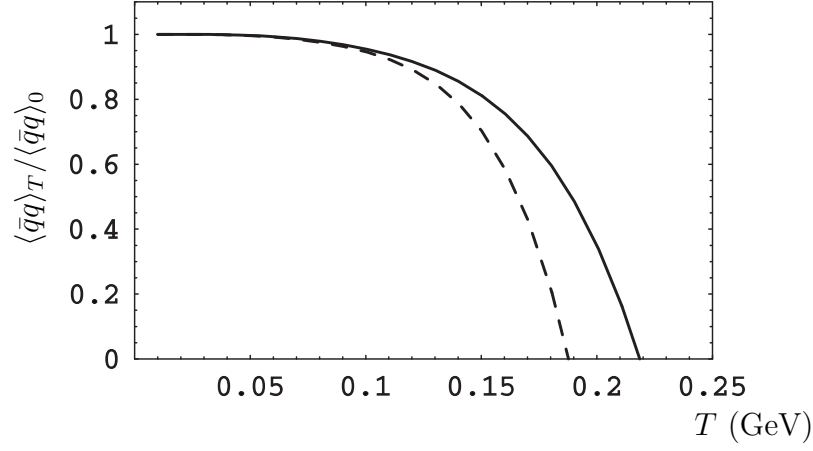


Figure 3: Quark condensate $\langle \bar{q}q \rangle_T / \langle \bar{q}q \rangle_0$ as a function of temperature. Solid line – zero temperature hadron spectrum; dashed line – spectrum with temperature shift taken into account, $\chi_T / \chi_0 = 0.84$.

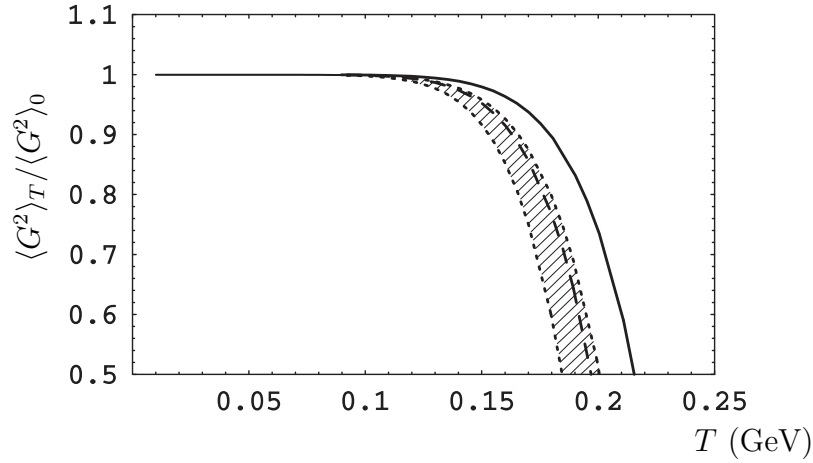


Figure 4: Gluon condensate $\langle G^2 \rangle_T / \langle G^2 \rangle_0$ as a function of temperature. Solid line – zero temperature hadron spectrum; dashed line – spectrum with temperature shift taken into account, $\chi_T / \chi_0 = 0.84$; $\langle G^2 \rangle_0 = 0.87 \text{ GeV}^4$ [26]. Shaded area – uncertainty due to zero-temperature gluon condensate value ($\langle G^2 \rangle_0 = 0.5 \div 1.0 \text{ GeV}^4$).

Strictly speaking, one has to find change of gluon condensate in a self-consistent way with the use of effective dilaton lagrangian at $T \neq 0$ and taking into account shift of hadron masses (see [22]). However, numerical calculations show that up to temperatures $T \sim m_\pi$ gluon condensate decreases very slowly, and at $T = m_\pi$, $\Delta\langle G^2 \rangle_T \approx 0.02\langle G^2 \rangle_0$. As temperature grows, gluon condensates drops abruptly and changes by $\sim 50\%$ in a small temperature region $\Delta T \sim 50$ MeV. Accordingly, we present calculations with temperature shift of hadron masses being 16% ($\chi_T/\chi_0 = 0.84 \simeq (0.5)^{1/4}$). Note, that even if temperature shift of m_h is not taken into account, quark condensate and half of gluon condensate evaporate at the same temperature $T \sim 215$ MeV.

5. In the present paper we have studied nonperturbative QCD vacuum with two light quarks at finite temperature in the framework of hadron resonance gas model. We have found temperature dependencies of quark and gluon condensates in the confined phase, and it was shown that quark condensate and one half of gluon condensate (chromo-electric component) evaporate at the same temperature, which corresponds to quark-hadron phase transition. This fact confirms picture of magnetic confinement, i.e. that chromo-electric condensate vanishes, while chromo-magnetic condensates almost does not change at the phase transition [3, 5, 6, 7]. The energy density of hadron resonance gas at transition temperature is $\varepsilon_h(T_c) \sim 1.5$ GeV/fm³. When temperature shift of hadrons masses is taken into account critical temperature is $T_c \simeq 190$ MeV.

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